

# Scaling Arguments

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The following statement is false. The preceding statement is true.

## Class Discussion

King-Kong. Alice in Wonderland. Bone cross-section, wing size, food.

## Warm Up

**Exercise 1.** Every camper gets their own bowl of soup. Every two campers get one bowl of spaghetti to share. Every three campers get one bowl of salad to share. All campers are required to have their own helping of salad, spaghetti, and soup. How many campers were there if the cook had to prepare 55 bowls of food?

**Exercise 2.** How many times do the clock hands form a 90-degree angle in the course of 12 hours?

**Exercise 3.** Which is the sum of all digits of the decimal representation of  $5^{16} \cdot 16^5$ ?

**Exercise 4.** What was the highest mountain in the world before Mount Everest was discovered?

## Scaling

**Exercise 5.** Estimate your weight on a planet with the same density as Earth and which diameter is twice smaller than the Earth's diameter.

**Exercise 6.** Estimate the number of gallons of water in all oceans.

**Exercise 7.** Suppose you found Yettie's tracks in Himalayas. The length of Yettie's foot is two feet. Assuming that Yettie should have the same pressure on the ground as humans, so it doesn't get stuck in the mud, how much taller is Yettie than an average human.

**Exercise 8.** Why would all creatures jump to approximately the same height? You are allowed to find the answer on Internet.

**Exercise 9.** Give an example of a movie where scaling is done unrealistically.

**Exercise 10.** You are shrunk so that your height is equal to the diameter of a dime (a ten-cent coin) and your mass is proportionally reduced so as to maintain your original density. Who is heavier, you or the dime?

## Competition Practice

**Exercise 11. 2004 AMC 8.** At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is  $2/5$ . What fraction of the people there are married men?

**Exercise 12. 2004 AMC 8.** Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

**Exercise 13. 2010 HMNT.** What is the remainder when  $(1 + x)^{2010}$  is divided by  $1 + x + x^2$ ?

## Challenge Problems

**Exercise 14.** Pooh went from his home to the Piglet's house for a sleepover. He left his home Friday at 6 pm and arrived at the Piglet's house at 8 pm. On Saturday, he left the Piglet's house at 6 pm, walked along the same path, and arrived home at 8 pm. Prove that there exists a point on his route such that he was at that point on Saturday at exactly the same time as on Friday.

**Exercise 15.** A king has four hats: three blue, and one red. He decides to test his three wise-men. He will put a hat on each man, so that they can see each other, but not their own hat. No communication between the men is allowed, except they do not have to say their colors at the same time. If any wise-man can figure out and say (out loud) to the king what colour hat he has on his head all three go free. What strategy the wise-men should use?