

# Envy-Free

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## Class Discussion

Envy-Free division for three people.

## Warm-Up

**Exercise 1.** Captain Cook made three voyages to the Pacific Ocean and died during one of them. Which one?

**Exercise 2.** If eggs are twelve cents a dozen, how much would 100 eggs cost?

**Exercise 3.** There are six glasses in a row. The first three are full of water, and the next three are empty. By touching only one glass how can you make them alternate between full and empty?

**Exercise 4.** A mile-long train is moving at sixty miles an hour when it reaches a mile-long tunnel. How long does it take the entire train to pass through the tunnel?

## Fair Division

**Exercise 5.** Alice and Bob are divorcing. They want to divide everything fairly. They own \$500,000 in cash. In addition, they own a house. Bob values the house at its market price of \$400,000. Alice loves the neighborhood and hates moving, so she values the house at \$440,000. They also own a vacation house that Bob uses for his fishing trips. Alice values the vacation house at \$200,000, while Bob at \$240,000. They also own a portrait of Alice's grandfather, which Bob values at 0, while Alice values at \$10,000. Suggest how they can divide their stuff fairly.

## Competition Practice

**Exercise 6.** Find a four digit number with distinct digits, such that the multiplication by 4 reverses the digits.

**Exercise 7. ARML.** How many zeroes do we write when we write all the integers from 1 to 256 in binary?

**Exercise 8. MATHCOUNTS.** The Sagebrush student council has 6 boys and 6 girls as class representatives. Two committees, each consisting of 2 boys and 2 girls, are to be created. If no student can serve on both committees, how many different combinations of committees are possible?

**Exercise 9.** How many positive integers less than 1000 are there such that the digits are in strictly increasing order?

**Exercise 10.** How many pairs of diagonals of a regular hexagon are parallel? Same question for an octagon and a decagon. Can you continue the sequence?

**Exercise 11.**  $N$  different circles are drawn in a plane. What is the maximum number of different points at which the circles can meet?

## Challenge Problem

**Exercise 12.** A fair division procedure is called *envy-free* if after the division no one wants to swap his/her part for someone else's. For three people it means each person believes s/he got at least one third and no one got more than s/he. Suggest an envy-free division strategy for four people. This is a difficult problem