

Continuity

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Class Discussion

Continuity arguments. The ham sandwich theorem (continuous and discreet).

Warm Up

Exercise 1. Feed, good, and kiss are 3 examples of words with double letters. Use the clues and fill in the blanks to find words with consecutive double letters. Remember, these words have 2 sets of double letters in a row!

1. It's a bird! It's a plane! It's a... B _ _ _ _ N
2. Body modification. T _ _ _ _ O
3. A Group of people. C _ _ _ _ _ _ E

Exercise 2. The blanks in the following sentences will be filled in with three different homonyms (words that are spelled differently but sound alike) to make valid sentences. The dashes indicate the number of letters in the words. Can you fill in the blanks?

1. The cut on his _ _ _ _ won't _ _ _ _ in time for the race, so _ _ ' _ _ have to drop out.
2. The man was so upset about being _ _ _ _ that he regularly _ _ _ _ _ himself up on the bed and _ _ _ _ _ his eyes out.
3. I couldn't _ _ _ _ _ any of the _ _ _ _ _ in the flower shop, because for some strange reason I had 50 _ _ _ _ _ crammed up my nose.
4. A bloodthirsty pirate will wander the _ _ _ _ and essentially _ _ _ _ _ everything he _ _ _ _ .

Continuity

Exercise 3. The soccer match ended with the score 8:5. Prove that there was a moment when the winning team had the same number of goals that the losing team had yet to score.

Exercise 4. There are 2000 points on a plane. Is it possible to draw a line so that there are 1000 points on each side?

Exercise 5. 10 pairs of boots line themselves up so that pairs are all mixed. Is it true that you can always find 10 boots in a row so that exactly half of them are left boots?

Exercise 6. Prove that it is possible to cut any convex polygon by two perpendicular lines in such a way that the four resulting figures have the same area.

Competition Practice

Exercise 7. 2000 AMC 10. When the mean, median, and mode of the list 10, 2, 5, 2, 4, 2, x are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real value of x ?

Exercise 8. 2000 AMC 10. Let f be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of z for which $f(3z) = 7$.

Exercise 9. 2000 AMC 10. If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) must be true?

1. All alligators are creepy crawlers.
2. Some ferocious creatures are creepy crawlers.
3. Some alligators are not creepy crawlers.

Exercise 10. Russian Linguistics Olympiad. Here are several Turkish words and phrases with their English translations: ada – an isle, adalar – isles, iki tas – two cups, adam – a man, otuz adam – thirty men, taslar – cups.

Pick the line of words from A to E that best translates these phrases into Turkish: thirty isles, men? (A) otuz adalar, adamlar; (B) otuz ada, adam; (C) otuz adalar, adam; (D) otuz ada, adamlar; (E) ikilar ada, adamlar.