Reflections

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My geometry teacher was sometimes acute, and sometimes obtuse, but always, he was right.

Class Discussion

M.C. Escher. Reflections. Reflections of a segment is a segment. Composition of reflections.

Warm Up

Exercise 1. "I guarantee", said the pet-shop salesman, "that this parrot will repeat every word it hears." A customer bought the parrot but found it wouldn't speak a single word. Nevertheless, the salesman told the truth. Explain.

Exercise 2. A solid, four-inch cube of wood is coated with blue paint on all six sides. Then the cube is cut into smaller one-inch cubes. These new one-inch cubes will have either three blue sides, two blue sides, one blue side, or no blue sides. How many of each will there be?

Exercise 3. A certain sheik named Hassan had eight horses. Four of them were white, three were black, and one was brown. How many of Hassan's horses can each say that it is the same color as another one of Hassan's horses?

Exercise 4. Abu, Ibn, and Hasib were suspects in a robbery. At the trial, they made the following statements, two of which are lies. Who committed the robbery?

- Abu: I didn't commit the robbery!
- Ibn: Hasib certainly didn't!
- Hasib: Yes, I did!

Reflections

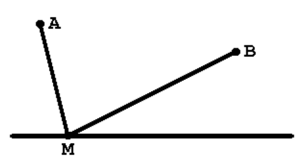
Exercise 5. Prove that a reflection of a circle is a circle.

Exercise 6. For the capital letters of the English alphabet find the letters that have a line of symmetry. Are there any letters that have more than one line of symmetry? (Strictly speaking, the symmetry of the letters depends on the font. But I can imagine a font in which the letter H has two lines of symmetry.)

Exercise 7. Prove that if a figure has exactly two intersecting lines of symmetry, then these lines are perpendicular.

Exercise 8. Can a figure have an infinite number of lines of symmetry? Can a bounded figure have an infinite number of lines of symmetry?

Exercise 9. You have line L and points A and B on the same side. Find a point M on line L such that the sum of distances $AM + MB$ is minimal.



Challenge Problems

Exercise 10. There is an infinite wall on the plane in the form of a straight line. You have the materials to build an extra piece of wall of any shape of length M. For some strange reason you want to build an enclosure of the maximal area and you can use the existing piece of wall. What shape should your wall be?

Exercise 11. Construct a quadrilateral $ABCD$ in which the diagonal AC bisects the angle A, given the lengths of the sides of the quadrilateral.