Calendar II

Tanya Khovanova

March 30, 2015

Class Discussion

Sara's Birthday: `http://blog.tanyakhovanova.com/?p=216`. Doomsday algorithm for any year. The cycle is 28 years within a century. Every 12 years add 1 day to the Doomsday. `http://blog.tanyakhovanova.com/2010/04/the-second-doomsday-lesson/`. Feb 29, 2000 was Tuesday. Feb 28, 1900, Wednesday.

Warm-up

Exercise 1. What is the largest amount of money one can have in coins without being able to make change for a dollar?

Exercise 2. In how many ways can you have 25 U.S. coins whose total is $1.00?

Exercise 3. I have an equal number of pennies, nickels and dimes. I also have some quarters which have the same value as the pennies, nickels and dimes combined. If I have no other coins, what is the fewest possible total positive number of coins I could have? What is the value of all the coins?

Exercise 4. You have ten boxes; each contains nine balls. The balls in one box weigh 0.9 kg; the rest weigh 1.0 kg. You have one weighing on an accurate scale to find the box containing the light balls. How do you do it?

Calendar

Exercise 5. Alex wants to celebrate his birthday at the moment the Earth passes the same point in its orbit when he was born. He was born on September 22, 1991 at 1 am. Assuming that a year is 365 days and 6 hours, create his celebration schedule.

Exercise 6. Calculate the days of the week for the following days: December 26, 1962, January 14, 1972, November 21, 2050, July 4, 1964, January 26, 2062, May 29, 1931, January 23, 1982, September 9, 1990.

Exercise 7. In the Gregorian calendar leap years are years divisible by 4, but not by 100, with the excpetion by 400. Assuming this calendar is exact, what is the length in days of one year?

Exercise 8. Assuming we will use the Gregorian calendar forever, what day of the month the 13 is more likely to fall on?

Competition Practice

Exercise 9. HMMT 1999. For what single digit n does 91 divide the 9-digit number $12345n789$?

Exercise 10. HMMT 1999. How many ways are there to cover a 3×8 rectangle with 12 identical dominoes?

Challenge Problems

Exercise 11. Eight coins weighing $1, 2, \ldots, 8$ grams are given, but which weighs how much is unknown. Baron Münchhausen claims he knows which coin is which; and offers to prove himself right by conducting one weighing on a balance scale, so as to unequivocally demonstrate the weight of at least one of the coins. Is this possible, or is he exaggerating?

Exercise 12. You have a balance scale and 12 coins, 1 of which is counterfeit. The counterfeit coin weighs less or more than the other coins. Can you determine the counterfeit in 3 weightings and tell if it is heavier or lighter. In this more difficult version you have to discribe your three weighings in advance. That is, the next weighing can not depend on the previous weighing.