

AMC Strategy. Percentages

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January 27, 2014

Class Discussion

Time for reading. Change in the problem difficulty. Advantages of correcting a problem. Checking versus continuing. Percentages.

Warm-Up

Exercise 1. Calculate:

$$1,000,000 - (1,000,000 - (1,000,000 - (1,000,000 - (1,000,000 - 999,999))))).$$

Exercise 2. Sergei had 5 friends over for his birthday party. For his first friend he cut $1/6$ of the cake, for the second friend $1/5$ of the rest, for the third friend $1/4$ of the new leftover cake, for the fourth friend — $1/3$ of what was left at the end. The last piece Sergei divided equally between himself and his last friend. Who got the biggest piece?

Competition Practice

Exercise 3. You have 24 pounds of nails. You have a balance scale without weights. How can you measure 9 pounds?

Exercise 4. Divide 7 full, 7 half-empty and 7 empty jars of honey between 3 people, so that every one gets the same amount of honey and the same number of jars.

Exercise 5. Calculate:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365}.$$

Exercise 6. 2006 AMC 10B, Problem 13. Joe and JoAnn each bought 12 ounces of coffee in a 16-ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

Exercise 7. 2006 AMC 10A, Problem 4. A digital watch displays hours and minutes with am and pm. What is the largest possible sum of the digits in the display?

Percentages

Exercise 8. In the first year after I bought a house its price increased by 100%. In the next year it dropped 50%. How did the price change since I bought the house?

Exercise 9. The Math Club, where Mike goes, is more than 93% girls. What is the smallest possible number of participants at the club?

Exercise 10. Mike is much shorter than Bob. Mike's steps are 20% shorter, but Mike makes 20% more steps than Bob at the same time. Who is walking faster?

Exercise 11. Alice and Bob sat on a bench to have lunch. Alice had 3 hot dogs and Bob had 4 hot dogs. Their friend Carl joined them, but Carl forgot his lunch at home. The three friends divided their hot dogs equally and after the lunch Carl paid \$7. How should Alice and Bob divide the money?

Exercise 12. A family went out. Alice had three times as much money as her brother Bob did. Their father had three times as much money as Alice, and their mother had three times as much money as the father. Altogether they had \$1,000. How much money did Alice have?

Challenge Problems

Exercise 13. Tigger, Piglet, and Pooh are at Pooh's house. They are going to Kanga's house which is 33 miles away. They have a 2-seat scooter which rides at 25 miles per hour with 1 rider on it; or, at 20 miles per hour with 2 riders. Each of the 3 friends walks at 5 miles per hour. Prove that all 3 of them can make it to Kanga's house in 3 hours.