Proofs Without Words

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Teacher: What are whole numbers?
Student: Like 0, 6, 8, 9.
Teacher: And what about 10?
Student: It is half-whole, 1 doesn't have a hole.

Class Discussion

Proofs without words.

Warm-Up

Exercise 1. Two friends played chess against each other for four hours. How many hours each of them played chess for?

Exercise 2. Ten kids from AMSA went on a tour of Greece. During the tour they visited 20 museums. How many museums did each kid go to?

Exercise 3. It takes 3 minutes to boil 3 eggs. How long will it take to boil 5 eggs?

Competition Practice

Exercise 4. 2002 AMC 10A. Problem 15. The digits 1, 2, 3, 4, 5, 6, 7, and 9 are used to form four two-digit prime numbers, with each digit used exactly once. What is the sum of these four primes?

Exercise 5. 2002 AMC 10B. Problem 6. For how many positive integers n is $n^2 - 3n + 2$ a prime number?

Exercise 6. 2002 AMC 10B. Problem 7. Let n be a positive integer such that $1/2 + 1/3 + 1/7 + 1/n$ is an integer. What is n?

Exercise 7. 1983 AIME. Let a_n equal $6^n + 8^n$. Determine the remainder upon dividing a_{83} by 49.

Exercise 8. Prove that the product of four consecutive integers plus 1 is a square.

Exercise 9. 2005 AMC 8, Problem 14. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

Exercise 10. AMC. I have a bag with coins and beads. Each object in the bag is either silver or gold. Thirty percent of the objects in the bag are beads and half of the coins in the bag are gold. What percent of the objects are silver coins?

Exercise 11. AMC. 1000 unit cubes (1 x 1 x 1 cubes) are glued together to form a 10 x 10 x 10 cube. At most how many of these unit cubes are visible from a single point in space?

Exercise 12. AMC. For any positive integer n, let $O(n)$ be the sum of the odd digits of n and $E(n)$ be the sum of the even digits of n. Let

$$x = O(1) + O(2) + O(3) + \cdots + O(100)$$

and

$$y = E(1) + E(2) + E(3) + \cdots + E(100).$$

Find x and y.

Challenge Problems

Exercise 13. All natural numbers are written one after the other, starting with 1. What digit occupies 206,788th place?

Exercise 14. Invent a way to continue the Pascal's triangle up.