

# AMC Preparation. Guessing

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## Class Discussion

AMC rules: 25 questions, 75 minutes, multiple choice out of 5. Each correct answer scores 6 points, a blank scores 1.5 points and an incorrect answer scores 0 points. Calculators are not allowed. Students who score 100 or above or finish in the top 5% on this AMC 12 or students who score 120 or above or finish in the top 2.5% on this AMC 10 will qualify for AIME. Students are allowed to take both contests.

NACLO Olympiad (January 30): <http://www.naclo.cs.cmu.edu/>

## Warm-Up

**Exercise 1.** Those of you who know anything about Catholicism, do you happen to know if the Catholic Church allows a man to marry his widow's sister?

**Exercise 2.** A certain street contains 100 buildings. A sign-maker is called to number the houses from 1 to 100. He has to order numerals to do the job. Without using pencil and paper, can you figure out in your head how many 9's he will need?

## AMC Preparation

**Exercise 3.** Peter has to transfer a head of cabbage, a goat and a wolf from one bank of a river to the other bank. But there is only one extra seat available on his boat. Furthermore, if the goat and the cabbage stay together as Peter is leaving on his boat, the goat will eat the cabbage. And if the wolf and the goat stay together as Peter is leaving, the wolf will eat the goat.

**Exercise 4.** Little Red Riding Hood who lives in Belmont went to visit her grandmother in Waltham. She departed at dawn and walked with a constant speed. Her grandmother's neighbor Nif-Nif decided to visit his brother Naf-Naf who is by accident a neighbor of Little Red Riding Hood. Nif-Nif used the same road as Little Red Riding Hood and he also walked with a constant speed. He also departed at dawn. At noon they both passed a sleeping wolf at the side of the road. They greeted each other, ignored the wolf and continued on their ways. At 4PM Little Red Riding Hood reached her grandmother and at 9PM Nif-Nif arrived at his brother's place. At what time was dawn?

**Exercise 5.** Parents of two kids, Mike and Tom, want to hire Tanya as a private tutor. Tanya gave the parents an old AMC 10 test and asked them to give it as a practice test to their kids. Mike got 25% of the questions. Tom's parents removed the multiple choice answers from the test and gave Tom just the questions. Tom got 20% of the questions. Tanya had time for only one kid. Whom did she prefer, Mike or Tom? How do you know that this problem is a fictional story?

**Exercise 6.** There are 5 heads and 14 legs in a family. This family has only dogs as pets. How many dogs are there in this family?

**Exercise 7.** Prove that  $11^{10} - 1$  is divisible by 100.

**Exercise 8.** Test for divisibility by 17. Subtract five times the last digit from the remaining leading truncated number. If the result is divisible by 17, then so was the first number. Apply this rule over and over again as necessary. Prove that this test works.

**Exercise 9.** Mike has 7 different magic cards for exchange and Tom has 9. In how many ways can they exchange one card for another? What if they exchange two cards for two cards?

## Challenge Problems

**Exercise 10.** Three Tajik sentences in Russian transliteration with their translations are below:

- дуусти хуби ҳамсоѝи шумо — a good friend of your neighbor
- ҳамсоѝи дуусти хуби шумо — a neighbor of your good friend
- ҳамсоѝи хуби дуусти шумо — a good neighbor of your friend

Your task is to assign a meaning to each out of four used Tajik words.