

Binary Numbers

Tanya Khovanova

October 21, 2013

There are 10 kinds of people in the world, those who understand binary, and those who don't.

Class Discussion

Binary numbers trick. Sorting with binary numbers, reversing.

Warm-Up

Exercise 1. We have 10 fingers on our hands. How many fingers are there on 10 hands?

Exercise 2. Mike and Tom went to a yard sale and wanted to buy a Yoda toy. Mike needed 10 more cents to buy the toy and Tom needed 1 more cent. They put their money together and they still didn't have enough. How much was Yoda?

Exercise 3. Can you replace the stars in the equation $1 \star 2 \star 3 \star \dots \star 10 = 0$ with pluses and minuses to get a correct equality?

Binary Numbers

Exercise 4. There are weights of 1g, 2g, 4g, and so on (all powers of 2), and there could be several weights of the same weight. The weights are put on two pans of the balance scale so that the scale balances. It is known, that the weights on the left pan are all different. Prove that the number of weights on the right pan is not less than the number of weights on the left pan.

Exercise 5. A common shuffling technique is called the *riffle* shuffle, in which half of the deck is held in each hand with the thumbs inward, then cards are released by the thumbs so that they fall to the table interleaved. Given a deck of sixteen cards in order what is the minimum number of riffle shuffles that is needed to put the deck in the reverse order? How? Prove that you cannot do it faster.

Competition Practice

Exercise 6. How many positive integers below 1000 are not divisible by 3 or 5? Another question: how many numbers are not divisible by 2 or 3 or 5?

Exercise 7. Find the sum of all four-digit even positive integers which you can write using the digits 0, 1, 2, 3, 4, 5 (you are allowed to reuse digits).

Exercise 8. MAML 2011-2012. This Olympiad has 25 questions. Correct answers are worth 6 points. Omitted questions are worth 2 points. Incorrect answers score 0 points. How many different scores are possible on this Olympiad?

Exercise 9. MAML 2011-2012. $7+9+11+13+\dots$ is an arithmetic series involving consecutive odd integers. $21+23+25+27+\dots$ is another arithmetic series involving consecutive odd integers. Let $S = 7 + 9 + 11 + 13 + \dots + s_k$ and $T = 21 + 23 + 25 + \dots + t_j$. For some ordered pairs (s_k, t_j) , $T - S = 34$. Compute $M - m$, where M denotes the maximum value of $s_k + t_j$ and m denotes the minimum value of $s_k + t_j$.

Challenge Problems

Exercise 10. You have a balance scale and 12 coins, one of which is counterfeit. The counterfeit weighs less or more than the other coins. Can you determine the counterfeit in 3 weighings, and tell if it is heavier or lighter?

If you need to just find the fake coin and do not need to tell if it is heavier or lighter, what is the maximum number of coins you can process in 3 weighings?