Sorting Algorithms

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A programmer sometimes wakes up in the middle of the night and feels thirsty. Soon, he thinks of a solution. He will leave 2 glasses next to his bed. One full of water, in case he wakes up, and he is thirsty, and one empty glass, in case he wakes up, and he is not thirsty.

Class Discussion

Sorting algorithms: Bubble sort, Insertion sort, Merge Sort, Quicksort.

Warm-Up

Exercise 1. Peter had ten cows. All but nine died. How many cows are left?

Exercise 2. Find the only two digit number n such that 2^n ends with n.

Exercise 3. Find the smallest value the second smallest angle of a convex hexagon with all integer degrees can have.

Exercise 4. Find the largest integer n such that there exists a Platonic solid with n vertices, a Platonic solid with n edges, and a Platonic solid with n faces.

Algorithms

Exercise 5. Multiplications are expensive. You need to program a function to calculate x^{17} when x is given. What is the smallest number of multiplications that you need? What will you do for the power 35? For any power?

Exercise 6. What is wrong with the following program that tries to swap two elements a and b if they are in a decreasing order?

Listing 1: Swapping Two Elements Wrong Code

```
if(a > b){
        //swap the elements!
        a = b;
        b = a;
        }
```

Competition Practice

Exercise 7. HMMT 2013 Guts Round. 6 points. For how many integers $1 \leq k \leq 2013$ does the decimal representation of k^k end with a 1?

Exercise 8. HMMT 2013 Guts Round. 11 points. The lines $y = x$, $y = 2x$, and $y = 3x$ are the three medians of a triangle with perimeter 1. Find the length of the longest side of the triangle.

Exercise 9. HMMT 2013 Guts Round. 14 points. Sherry and Val are playing a game. Sherry has a deck containing 2011 red cards and 2012 black cards, shuffled randomly. Sherry flips these cards over one at a time, and before she flips each card over, Val guesses whether it is red or black. If Val guesses correctly, she wins 1 dollar; otherwise, she loses 1 dollar. In addition, Val must guess red exactly 2011 times. If Val plays optimally, what is her expected profit from this game?

Challenge Problem

Exercise 10. In your favorite programming language, write a program that, when run, will print out its own source code.