Miscellaneous 2012

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Class Discussion

In what base 2012! has more zeroes: 13, 14, 15, 16, 17? Order by base.

Warm-Up

Exercise 1. Two boys and two girls discussed a test answer.

- Dave said, "The answer is 9."
- Bob said, "The answer is a prime number."
- Kate said, "It is an even number."
- Ann said, "The answer is 15."

What is the answer if exactly one boy and one girl were right.

Exercise 2. There are 2000 people in the building. 3.7% of them have one earring. One half of the rest has two earrings, while the other half has no earrings. How many earrings are in the building? Did you do it by calculating percentages?

Exercise 3. When it's raining, the cat is either in the kitchen or in the basement. When the cat is in the kitchen, the mouse is in her nest, and the cheese is in the fridge. If the cheese is on the kitchen table and the cat is in the basement, then the mouse is in the kitchen. It is raining now and the cheese is on the kitchen table. Which of the following statements follows: a) the cat is in the kitchen, b) the mouse is in the nest, c) the cat is in the kitchen and the mouse is in the nest, d) the cat is in the basement and the mouse is in the kitchen.

Exercise 4. I have three containers with labels: Potatoes, Apples, and Fruits. I bought apples, bananas, and potatoes and put them into the three different container so that all the labels are wrong. What is in each container?

Competition Practice

Exercise 5. Can you solve the following problem without the exact calculation?

MAML 2011. The number $20000 - 17 \times 25$ is the product of three consecutive odd numbers, one of which is a perfect square, one of which is a perfect cube, and one of which is prime. What is the prime number?

Exercise 6. Solve the following problem and tell how the writer tried to confuse everyone.

MAML 2011. Which of the following is equal to $\frac{\sqrt{2^6+2^6+2^6+4^3}}{\sqrt[3]{6\times15\times300}}$?

A) 1/15 B) 2/15 C) 8/15 D) 4/15 E) 16/15

Exercise 7. Invent a theorem that states when the number in odd base is odd, and solve the following problem.

MAML 2011. The product of the four-digit natural numbers $P = 4A32_{(base\ 9)}$ and $Q = A2A1_{(base\ 11)}$ is odd. For how many values of the digit A is this true?

Exercise 8. MAML 2011. If 2011! is converted from base 10 to base 13, the base 13 representation will end in a string of k zeros. Compute k.

Challenge Problems

Exercise 9. A big candle lasts one hour and costs 60 rubles. A small candle lasts 11 minutes and costs 11 rubles. Can you measure a minute by spending not more than a) 200 rubles, b) 150 rubles?

Exercise 10. Let n be a fixed integer. Kostya has a black box, such that if you put in exactly $2n + 1$ coins of distinct weights, the box will expose the coin of median weight. The Baron gave Kostya $4n + 1$ coins of distinct weights and told him which coin has the median weight. Can Kostya check that the Baron is right, using the box not more than $n + 2$ times? If you can't solve it for any n, try $n = 3$.