

Integers in Computers

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March 25, 2013

A keyboard for sale: only one key doesn't work.

Class Discussion

Representing integers in computers. Two's complement. The largest integer. The smallest integer. Addition, subtraction, multiplication.

Warm-Up

Exercise 1. A bus driver was heading down a street in Boston. He went right past a stop sign without stopping, he turned left where there was a no-left-turn sign and he went the wrong way on a one-way street. Then he went on the left side of the road past a cop car. Still, he didn't break any traffic laws. Why not?

Exercise 2. There are only two triangular numbers such that their squares are also triangular. Find them.

Exercise 3. Find the greatest common factor of the indices of the Fibonacci numbers divisible by 13.

Exercise 4. Find the largest common divisor of numbers of the form $p^2 - 1$ for primes p greater than three.

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Exercise 5. Write a program that multiplies two positive integers and outputs a negative integer. If integers are represented with 32 bits, give an example of two integers that when multiplied by a computer will output -2 .

Exercise 6. Write a program that sums powers of two and outputs -1 . If integers are represented with 32 bits, what powers of two you should add up.

Competition Practice

Exercise 7. USAMO 1990. A certain state issues license plates consisting of six digits (from 0 through 9). The state requires that any two plates differ in at least two places. (Thus the plates $\boxed{027592}$ and $\boxed{020592}$ cannot both be used.) Determine, with proof, the maximum number of distinct license plates that the state can use.

Exercise 8. Math Kangaroo 2013. How many pairs (x, y) of integers with $x \leq y$ exist such that their product equals 5 times their sum?

Exercise 9. Math Kangaroo 2013. On the island of Knights and Knaves there live only two types of people: Knights (who always speak the truth) and Knaves (who always lie). I met two men who lived there and asked the taller man if they were both Knights. He replied, but I couldn't figure out what they were, so I asked the shorter man if the taller was a Knight. He replied, and after that I knew which type they were.

Can you figure out who were the men?

Challenge Problem

Exercise 10. There is an array containing all the integers from 1 to n in some order, except that one integer is missing. Suggest an algorithm for finding the missing number in $O(n)$ time and $O(1)$ space.