Floating Point

Tanya Khovanova

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My computer always beats me in chess. In revenge, I always beat it in a boxing match.

Class Discussion

Representation of doubles: sign bit, 11 exponent bits, 52 mantissa bits. Exponent bias is -1023. Zeros and subnormals. Infinity and NaNs.

Warm-Up

Exercise 1. How can you throw a ball as hard as you can and have it come back to you, even if it doesn't bounce off anything? There is nothing attached to it, and no one else catches or throws it back to you.

Exercise 2. Find the number of positive integers n for which the sum of the n smallest positive integers evenly divides $18n$.

Exercise 3. Find the positive difference between the two prime numbers that do not share a unit digit with any other prime number.

Exercise 4. What is the number of 5-digit palindromic integers in base 5?

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Exercise 5. Find two float or double numbers x and y so that a Java program that calculates their sum doesn't output the exact sum. Do it by experiment or theoretically. Explain.

Exercise 6. Find out the next double after 0.3. Either experiment with programming or figure it out theoretically.

Competition Practice

Exercise 7. Moscow Olympiad 2012. Alexey wrote 5 integers on the blackboard: roots and coefficients of a quadratic equations. Boris erased one of the integers. Find with proof the erased number if the remaining numbers are -5, 2, 3, 4.

Exercise 8. Moscow Olympiad 2012. There are $2n$ pears that are placed in a row. The weights of any two neighboring pears differ by not more than 1 gram. Prove that you can place the pears into n bags (exactly 2 pears into one bag) and put the bags in a row in such a way that any two neighboring bags differ by not more than 1 gram.

Challenge Problem

Exercise 9. The Grand Master vowed to set up a truly fair test to reveal the best logician amongst his three friends (without giving an advantage to any one of them). He showed the three men 5 hats: two white and three black. Then he turned off the lights in the room and put a hat on each logician's head. The logicians would need to guess the color of their own hat. After that the old sage hid the remaining two hats, but before he could turn the lights on, the logicians announced the correct colors of their hats. What was their reasoning?

Exercise 10. Given a double number d what is the greatest double number x such that $(d - x) == d$ could be true? Suppose d is any double number, what is the greatest number x such that $(d - x) == d$ could be true?