Euler Formula

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Class Discussion

Euler formula: $E + 2 = V + F$. Proof. Euler formula for a sphere and a torus. Walter Wick's Optical Tricks.

Warm-Up

Exercise 1. Which letter comes next in the series? S, M, H, D, W, M, ...?

Exercise 2. If 2 hours ago it was as long after one o'clock in the afternoon as it was before one o'clock in the morning what time would it be now?

Exercise 3. Between 1000 and 2000 you can get each integer as the sum of several nonnegative consecutive integers. For example, $147 + 148 + 149 + 150 + 151 + 152 + 153 = 1050$. There is only one number that you cannot get. What is this number?

Exercise 4. One day three Greek philosophers settled under the shade of an olive tree, opened a bottle of Retsina, and began a lengthy discussion of the Fundamental Ontological Question: Why does anything exist? After a while, they began to ramble. Then, one by one, they fell asleep. While the men slept, three owls, one above each philosopher, completed their digestive process, dropped a present on each philosopher's forehead, the flew off with a noisy "hoot." Perhaps the hoot awakened the philosophers. As soon as they looked at each other, all three began, simultaneously, to laugh. Then, abruptly, they stopped laughing. Why?

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Exercise 5. There are several islands connected by bridges so that you can get from any island to any other island. A tourist walked through all the islands crossing each bridge exactly once. He visited the Triad island three times. How many bridges start from the Triad if:

- the tourist started and ended not on the Triad?
- the tourist started on the Triad and ended elsewhere?
- the tourist started and ended on the Triad?

Exercise 6. Someone put 20 dots inside a square and connected the dots with each other and with the vertices of the square in such a way that the square became divided into triangles. How many triangles are there?

Exercise 7. Prove the following inequality for a connected planar graph with at least three vertices: $E \leq 3V - 6$.

Exercise 8. Every edge of a complete graph with 11 vertices is colored either blue or red. Proof that either, the "red" or the "blue" graph is not planar.

Exercise 9. Prove that the utility graph is not planar by using the Euler formula. The utility graph is the graph connecting three cottages to gas, water and electric companies.

Challenge Problems

Exercise 10. Five pirates of different ages have a treasure of 100 gold coins. On their ship, they decide to split the coins using this scheme:

The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it. If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain.

As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard. Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?