Coloring Method

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Class Discussion

Problem. Suppose a standard 8x8 chessboard has two diagonally opposite corners removed, leaving 62 squares. Is it possible to place 31 dominoes of size 2x1 so as to cover all of these squares?

Problem. Is it possible to place tiles of size 4x1 so as to cover the board of size 10x10?

Warm-Up

Exercise 1. A math professor wrote four letters to four colleagues and put corresponding addresses on four envelopes. Then he started thinking about a math problem and put the letters into different envelopes randomly. What is the probability that exactly three letters are put correctly?

Exercise 2. Simplify $\sin(\alpha) \times \sin(\beta) \times \sin(\gamma) \times \ldots \times \sin(\omega)$.

Exercise 3. A town has two hospitals: one large and one small. Today one of these two hospitals has 60% of boys among newborns. Assuming there is an equal number of boys and girls born every year in the United States, which hospital is more probably to be the one with 60% boys?

Exercise 4. I have a pile of paper that contains exactly 100 sheets, and I need exactly 80. I can count 10 sheets of paper in 10 seconds. How much time will it take me to get what I want?

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Exercise 5. There is one chip on each cell of a 9 by 9 board. I want to move each chip to a neighboring cell (horizontally or vertically). Can I do it so that at the end I will again have one chip on each cell?

Exercise 6. A crocodile is a new chess piece that in one move jumps m cells in one direction (vertical or horizontal) and then n cells in a perpendicular direction. Is it possible to color an infinite board in two colors so that any crocodile jump always changes color?

Exercise 7. Can you put 13 bricks of size $1 \times 1 \times 2$ together to form a cube of size $3 \times 3 \times 3$ with a $1 \times 1 \times 1$ hole in its center?

Exercise 8. The plane is colored in three colors. Prove that there are two points of the same color at a distance 1.

Competition Practice

Exercise 9. 2005 AMC12. The graph of the polynomial $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ has five distinct x-intercepts, one of which is at $(0, 0)$. Which of the coefficients a, b, c, d, e cannot be zero?

Challenge Problem

Exercise 10. On an island there lived three kinds of people: liars who always lie, truth-tellers who always tell the truth, and alternators, who always alternate between the lie and the truth. Once three strangers from this island met and had the following conversation:

- A: I do not know whether there are liars among us.
- B: I do not know whether there are truth-tellers among us.
- C: I do not know whether there are alternators among us.
- A: I even do not know whether there are liars among you.
- B: I even do not know whether there are truth-tellers among you.
- C: I even do not know whether there are alternators among you.

Who is who?