Combinatorics Review

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Class Discussion

Chosing k objects of n types.

	repetition allowed	repetition not allowed
order matters	n^k	$\frac{n!}{(n-k)!}$
order doesn't matter	$\binom{n+k-1}{k}$	$\binom{n}{k}$

Derangements.

Warm-Up

Exercise 1. A clerk in the butcher shop is 5'10" tall. What does he weigh?

Exercise 2. In a family photo you see: one grandmother, two mothers, one father, two sons, one grandson, one mother-in-law, one daughter-in-law, one husband, one wife. What is the fewest number of people possible in the photo?

Exercise 3. Continue the sequence: 3, 3, 5, 4, 4, 3, 5, 5, 4, ...

Combinatorics

Exercise 4. The bad guy discovered that Tanya's password starts with 2 capital letters followed by 3 digits and then by 2 lower-case letters. How many possibilities are there for Tanya's password?

Exercise 5. Tanya wants to pick an HMNT team of six people from the total of 43 students. In how many ways can she do that? What if she wants to pick two teams?

Exercise 6. Tanya has 12 students chosen for HMNT. She wants to pick one of them as a corresponding author and a different one as one responsible for arranging transportation. In how many ways can she do that? What if two roles are allowed to be played by the same person?

Exercise 7. HMNT organizers need to buy 40 pizzas for the competition. There are 8 possible flavors. In how many ways can they buy pizzas?

Competition Practice

Exercise 8. HMMT. How many nonempty subsets of $\{1, 2, 3, \ldots, 12\}$ have the property that the sum of the largest element and the smallest element is 13?

Exercise 9. HMMT. For how many ordered triples (a, b, c) of positive integers less than 10 is the product abc divisible by 20?

Challenge Problems

Exercise 10. AIME. A hotel packed a breakfast for each of three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese and fruit rolls. The preparer wrapped each of the nine rolls, and, once they were wrapped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. Find the probability that each guest got one roll of each type.