

# Bases

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## Class Discussion

Base systems.

## Warm-Up

**Exercise 1.** Find the largest number consisting of distinct digits such that, in its English pronunciation, all the words start with the same letter. Find the largest number consisting of the same digit such that, in its English pronunciation, all the words start with different letters.

**Exercise 2.** I thought of a 6 digit number and told the sum of its digits to Sergei. After that Sergei knew my number. What was it?

## Bases

**Exercise 3.** Can a number written only with fours be divisible by a number that is written only with threes? What if we switch three and four in the condition?

**Exercise 4.** Find the largest six digit integer such that every digit starting with the third one is the sum of the two preceding digits. Find the largest integer such that every digit starting with the third one is the sum of the two preceding digits.

**Exercise 5.** Find a two digit number that is twice the product of its digits.

**Exercise 6.** Find the rule to check divisibility by 3 for binary numbers.

**Exercise 7.** Find the smallest base such that a number in this base is divisible by 5 iff (if and only if) the sum of the digits is divisible by five and a number in this base is divisible by 7 iff the last two digits form a number divisible by 7.

**Exercise 8.** Tom and Jerry are riding in a train and are counting the poles. Tom doesn't like the letter  $R$ , so he skips numbers that contain letter  $R$ . Jerry doesn't like the letter  $X$ , so he skips numbers that contain letter  $X$ . Tom counted the last pole as 99. What number will Jerry get for this pole?

**Exercise 9.** I am showing a magic trick. I have nine cards. You choose one and shuffle it back in. I deal the cards one by one into three columns. I ask you in which column is your card. When I collect the cards I put the column you chose in the middle. Then I deal again and ask again, and again put your column in the middle. Before the last deal I am telling you where your card will be. Where will it be? Explain.

## Challenge Problems

**Exercise 10.** What is the smallest number of weights of integer weights do you need so you can measure any number of grams from 1 to 100 inclusive on a balance scale. You can put weights on both pans.