Quadratics

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Class Discussion

$x^2 - 5x + 6$, $x^2 - 12x - 540$ — use divisibility. $3t^2 + 11t + 10 = 0$. Find the ratio x/y if $x^2 - 5xy + 6y^2 = 0$.

Warm-Up

Exercise 1. There are three people (Alex, Brook and Cody), one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. Alex says: "Cody is a knave." Brook says: "Alex is a knight." Cody says: "I am the spy."

Who is the knight, who the knave, and who the spy?

Exercise 2. You need to take socks from a drawer in a very dark room. You are told that the drawer contains 6 blue socks, 5 red socks, and 10 white socks. What is the smallest number of socks you need to take before you can be sure to have at least one matching pair?

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Exercise 3. AHSME. How many integers x satisfy the equation: $(x^2 - x - 1)^{x+2} = 1$?

Exercise 4. Consider the set of parabolas defined by $y = x^2 + px + q$, where $p + q = 2011$. Prove that all those parabolas pass through the same point on the plane.

Exercise 5. AHSME. The sum of the squares of the roots of the equation $x^2 + 2hx = 3$ is 10. Find $|h|$.

Exercise 6. The quadratics $y = ax^2 + bx + c$ doesn't have real roots and $a + b + c > 0$. Is c positive or negative?

Exercise 7. AMC 10. Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + 1/b$ and $b + 1/a$ are the roots of the equation $x^2 - px + q = 0$. What is q?

Exercise 8. Both roots of the equation $x^2 + px + q = 0$ are integers. Find p and q, given that p and q are prime numbers.

Exercise 9. For which a, one of the roots of the equation $x^2 - \frac{15}{4}x + a^3$ is a square of the other root?

Exercise 10. HMMT. Find all real solutions (x, y) of the system $x^2 + y = 12 = y^2 + x$.

Competition Practice

Exercise 11. 2006 USAMO. For a given positive integer k find, in terms of k, the minimum value of N for which there is a set of $2k + 1$ distinct positive integers that has sum greater than N but every subset of size k has sum at most $N/2$.

Challenge Problems

Exercise 12. Six segments are such that you can make a triangle out of any three of them. Is it true that you can build a tetrahedron out of all six of them?

Exercise 13. An ant is sitting on the corner of a brick. A brick means a solid rectangular parallelepiped. The ant has a math degree and knows the shortest way to crawl to any point on the surface of the brick. Is it true that the farthest point from the ant is the opposite corner?