

Pythagorean Theorem

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January 31, 2011

Einstein-Pythagorean equation $E = m(a^2 + b^2)$.

Class Discussion

Different proofs of the Pythagorean theorem.

Warm-up

Exercise 1. Show how you can plant 8 trees and end up with 6 rows of 3 trees each.

Exercise 2. Nine dots are arranged into a three by three grid. What is the smallest number of squares needed to separate each of the dots?

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Exercise 3. One of the legs of a right triangle is 10 meters longer than the other leg, and 10 meters shorter than the hypotenuse. Find the hypotenuse.

Exercise 4. Prove that in a right trapezoid, the difference of squares of the diagonals equals the difference of squares of the parallel sides.

Exercise 5. How many right triangles with sides of integer lengths are there such that one of the legs is 15?

Competition Practice

Exercise 6. 2003 AMC 10A. Simplify $\sqrt[3]{x\sqrt{x^3\sqrt{x^3\sqrt{x\sqrt{x}}}}}$.

Exercise 7. 2003 AMC 10A. The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

Exercise 8. 2003 AMC 10A. A point (x, y) is randomly picked from inside the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

Exercise 9. 2003 AMC 10A. The sum of three numbers is 20. The first is 4 times the sum of the other two. The second is seven times the third. What is the product of all three?

Exercise 10. 2005 AMC 10A. For each positive integer $m > 1$, let $P(m)$ denote the greatest prime factor of m . For how many positive integers n is it true that both $P(n) = \sqrt{n}$ and $P(n + 48) = \sqrt{n + 48}$?

Exercise 11. 2005 AMC 10A. In $\triangle ABC$ we have $AB = 25$, $BC = 39$, and $AC = 42$. Points D and E are on \overline{AB} and \overline{AC} respectively, with $AD = 19$ and $AE = 14$. What is the ratio of the area of triangle ADE to the area of the quadrilateral $BCED$?

Exercise 12. Linguistics Olympiad. The following list shows some numbers, written by words in the language of some Pacific island nation. Each next number is equal to the previous one plus 2. Can you determine what these numbers are?

- thabung ke nua lo
- thabung ke nua vak
- libenyita ke nua khasa
- libenyita ke nua kun
- libenyita ke nua thabung
- libenyita ke nua thabung ke nua lo
- libenyita ke nua thabung ke nua vak