

Magic Square

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A minus is half a plus, but every plus has its own minus...

Class Discussion

The Magic Square. The 15 game. Tic-tac-toe.

Warm-Up

Exercise 1. Nick names two different numbers a and b . Ryan changes two stars in the equation $* \cdot * = *$ for a and one star for b . If the resulting equality is true, Nick wins. Otherwise, Ryan wins. How should Nick play this game?

Exercise 2. Christine has six pairs of black socks and six pairs of white socks in her drawer. In complete darkness, and without looking, how many socks must she take from the drawer in order to be sure to get a pair that match?

The Magic Square

Exercise 3. Construct a four by four magic square.

Exercise 4. 2001 AMC 10.

x	24	w
18	v	y
25	z	21

In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of these numbers are represented by v , w , x , y , and z . Find $y + z$.

Exercise 5. What is the most/least optimal first move in the tic-tac-toe? Explain.

Exercise 6. Tim and Davio are playing tic-tac-toe. Tim decided to explore risks and started the game by putting a cross in the middle of a side. Davio grabbed the center. What next move should Tim avoid?

Competition Practice

Exercise 7. 2001 AMC 10. A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

Exercise 8. 2002 AMC 10-A. Points A , B , C , and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line, and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB .

Exercise 9. 2002 AMC 10-A. In trapezoid $ABCD$ with bases \overline{AB} and \overline{CD} , we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. What is the area of $ABCD$?

Exercise 10. 2002 AMC 10-A. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \dots, 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

Challenge Problems

Exercise 11. In the equation $ELM + ELM + \dots + ELM = PARK$ different letters mean different digits, and the same letter means the same digit. What is the maximum number of elms that could grow in the park?

Exercise 12. I will put a hat, either white or black, on the head of each of you. You can see others' hats, but you can't see your own hat. You are given 20 minutes. I will place at least one white hat and at least one black hat. At least one of you should tell me the colour of the hat on your head. You can't signal to others or give a hint or anything like that. You should say only WHITE or BLACK. You should decide on your strategy before the next class.