

Fair Division Review

Tanya Khovanova

May 2, 2011

Class Discussion

Envy-free division for three people.

Warm-Up

Exercise 1. If I start with \$10 and spend all but \$3, how much do I have left?

Exercise 2. Start with $x = 1$ and $y = 1$. It follows that $x = y$. Multiplying each side by x , we get: $x^2 = xy$. Subtracting y^2 from each side, we get:

$$x^2 - y^2 = xy - y^2.$$

Now we can factor each side: $(x + y)(x - y) = y(x - y)$. Then we can divide out the common term $(x - y)$: $x + y = y$. Substituting the given values, we get: $1 + 1 = 1$, or $2 = 1$.

Exercise 3. What is the next letter in the following sequence: O, T, T, F, F, S, S, E?

Probability

These problems are based on puzzles from the book by Mari Berrondo.

Exercise 4. Alice, Bob and Carl went hunting. Alice is the best shot — she hits the target with probability 3 out of 5. Next is Bob, who is successful with probability 3 out of 10. Carl hits with probability 1 out of 10. They saw a tiger and all made one shot at the same time. What is the probability that the tiger is hit?

Exercise 5. Three boys and two girls sat on a bench in a random order. What is more probable: that the two girls sat next to each other or that they didn't?

Exercise 6. The math team consists of three boys and three girls who like to hang out together. Each girl loves one of the boys and each boy loves one of the girls. If each person chooses the object of his/her affection randomly, what is the probability that they end up in a situation where no one's feelings are reciprocated?

Competition Practice

Exercise 7. AMC. I have 120 blocks. Each block is one of 2 different materials, 3 different colors, 4 different sizes and 5 different shapes. No two blocks are exactly the same in all four properties. I take two blocks at random. What is the probability the two blocks have exactly two of these four properties the same?

Exercise 8. MATHCOUNTS. A fair 6-sided die is tossed 8 times. The sequence of 8 results is recorded to form an 8-digit number. What is the probability that the number formed is a multiple of 8?

Exercise 9. A line segment is broken at two random points along its length. What is the probability that the three new segments form a triangle?

Challenge Problems

Exercise 10. We were giving a formal dinner party for ten (including ourselves) which is a number I always like because the host can sit at one end of the table and the hostess at the other, and still maintain the correct alternate male and female around the table. My wife was trying to work out the seating. "Tom and Jean have not been here to dinner before so they are the guests of honor. Tom must sit on my right and Jean on your right, but I don't know how I want to seat the others.:" "Well," I said, "I would like Janet on my left. I have a soft spot for her." "You can have her," replied my wife, "but I will not have her husband Jack next to me; I think he should be next to Mary Ann."

Since we do not place husbands and wives next to each other, this determined the seating of everyone, including Howard's wife Lois, and Mary Ann's husband Bill. Can you work out the seating arrangement?