

# The Cantor Set

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## Class Discussion

The Cantor Set, the Sierpinski triangle, Odd numbers in the Pascal's triangle.  
Binary tree movie by Vi Hart.

## Warm-Up

**Exercise 1.** What did you wear the day after yesterday?

**Exercise 2.** There is a pink house made of pink bricks, a blue house made of blue bricks, an orange house made of orange bricks. What color bricks is a greenhouse made of?

**Exercise 3.** What digit is the most frequent between the numbers 1 and 1,000 inclusive?

## Pascal's Triangle

**Exercise 4.** Derive a formula for the number of odd entries in row  $n$  of Pascal's triangle.

## Competition Practice. Moscow Math Olympiad 2011. 8th grade

**Exercise 5.** There were 6 seemingly identical balls lying at the vertices of the hexagon  $ABCDEF$ : at  $A$  — with a mass of 1 gram, at  $B$  — with a mass of 2 grams, ..., at  $F$  — with a mass of 6 grams. A hacker switched two balls that were at opposite vertices of the hexagon. There is a balance scale that

allows you to say in which pan the weight of the balls is greater. How can you decide which pair of balls was switched, using the scale just once?

**Exercise 6.** Peter was born in the 19th century, while his brother Paul was born in the 20th. Once the brothers met at a party celebrating both birthdays. Peter said, “My age is equal to the sum of the digits of my birth year.” “Mine too,” replied Paul. By how many years is Paul younger than Peter?

**Exercise 7.** Does there exist a hexagon which can be divided into four congruent triangles by a single line?

**Exercise 8.** Every straight segment of a non-self-intersecting path contains an odd number of sides of cells of a  $100 \times 100$  square grid. Any two consecutive segments are perpendicular to each other. Can the path pass through all the grid vertices inside and on the border of the square?

**Exercise 9.** Denote the midpoints of the non-parallel sides  $AB$  and  $CD$  of the trapezoid  $ABCD$  by  $M$  and  $N$  respectively. The perpendicular from the point  $M$  to the diagonal  $AC$  and the perpendicular from the point  $N$  to the diagonal  $BD$  intersect at the point  $P$ . Prove that  $PA = PD$ .

**Exercise 10.** Each cell in a square table contains a number. The sum of the two greatest numbers in each row is  $a$ , and the sum of the two greatest numbers in each column is  $b$ . Prove that  $a = b$ .