

Coins

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Question: How many sides does a box have?

Answer: Two — the inside and the outside.

Class Discussion

Doomsday quiz. Square breathing.

Warm-up

Exercise 1. Do the following standard phrases make sense?

- I fell head over heels.
- I could care less.
- I turned my life around 360 degrees.

Exercise 2. We have a simple balancing scale with 2 cups (and no extra weights), and a set of N coins, which all look identical. We know that there is exactly one fake coin among them, and it is a little bit lighter than a genuine coin. All genuine coins weight the same. For which N you can always find the fake coin in two weighings? What about three weighings? What about k weighings?

Exercise 3. We have a simple balancing scale with 2 trays (and no extra weights), and a set of N coins, which all look identical. We know that there is exactly one fake coin among them, and that it's either lighter, or heavier than a genuine coin. We would like to find out whether the fake coin is heavier or lighter than a genuine one, (note that we are not trying to find

the actual coin itself) but we are only allowed to use the scale 2 times. Find all numbers N for which this can be done.

Exercise 4. You have 2 glasses of the same size, one with coffee, the other with milk. First, take a spoonful of coffee, add it to milk, and mix well. Then, take a spoonful (same amount as before) of this mixture, add it back to coffee, and mix well. Is there more milk in the coffee now, or more coffee in the milk?

Exercise 5. Can a cube of cheese three inches on a side be cut into 27 one-inch cubes with five straight cuts? What if one can move the pieces prior to cutting?

Competition Practice

Exercise 6. HMNT 2008. What is the largest x such that x^2 divides $24 \cdot 35 \cdot 46 \cdot 57$?

Exercise 7. USAMO 2000. Find the smallest positive integer n such that if n squares of a 1000×1000 chessboard are colored, then there will exist three colored squares whose centers form a right triangle with sides parallel to the edges of the board.

Exercise 8. IMO 1976. Determine, with proof, the largest number which is the product of positive integers whose sum is 1976.

Challenge Problems

Exercise 9. You have a balance scale and 12 coins, 1 of which is counterfeit. The counterfeit weigh less or more than the other coins. Can you determine the counterfeit in 3 weightings, and tell if it is heavier or lighter?

Exercise 10. How many zeroes are at the end of $1000!$?