

Sequences. Other Homework problems

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Finish the problems from the class handout.

Competition practice

Exercise 1. 2005 AMC 8, Problem 14. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

Exercise 2. AMC. I have a bag with coins and beads. Each object in the bag is either silver or gold. Thirty percent of the objects in the bag are beads and half of the coins in the bag are gold. What percent of the objects are silver coins?

Exercise 3. AMC. 1000 unit cubes ($1 \times 1 \times 1$ cubes) are glued together to form a $10 \times 10 \times 10$ cube. At most how many of these unit cubes are visible from a single point in space?

Exercise 4. AMC. For any positive integer n , let $O(n)$ be the sum of the odd digits of n and $E(n)$ be the sum of the even digits of n . Let

$$x = O(1) + O(2) + O(3) + \cdots + O(100)$$

and

$$y = E(1) + E(2) + E(3) + \cdots + E(100).$$

Find x and y .

Challenge Problems

Exercise 5. Mr. House would like to visit his old friend Mr. Street, who is living in the main street of a small village. The main street has 50 houses divided into two blocks and numbered from 1 to 20 and 21 to 50. Since Mr. House has forgotten the number, he asks it from a passer-by, who replies “Just try to guess it.” Mr. House likes playing games and asks three questions:

- In which block is it?
- Is the number even?
- Is it a square?

After Mr. House has received the answers, he says: “I’m still doubting, but if you’ll tell me whether the digit 4 is in the number, I will know the answer!”. Then Mr. House runs to the building in which he thinks his friend is living. He rings, a man opens the door and it turns out that he’s wrong. The man starts laughing and tells Mr. House: “Your advisor is the biggest liar of the whole village. He never speaks the truth!”. Mr. House thinks for a moment and says “Thanks, now I know the real address of Mr. Street”. What is the address of Mr. Street?