Remainders

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Class Discussion

Remainders. Remainders of squares. Converting miles to kilometers using Fibonacci numbers.

Warm Up

Exercise 1. Prove that $6^{2n+1} + 1$ is divisible by 7.

Exercise 2. The population of the island of Pianosa is 100. Some of the inhabitants always lie, the others always tell the truth. Each islander worships one of three gods: the Sun god, the Moon god, or the Earth god. One day a visiting anthropologist asked each inhabitant the following questions:

1. Do you worship the Sun god?
2. Do you worship the Moon god?
3. Do you worship the Earth god?

There were 60 "yes" answers to the first question, 40 "yes" answers to the second question, and 30 "yes" answers to the third. How many liars live on the island?

Problem Set

Exercise 3. Prove that the remainder of a prime number modulo 30 is either prime or 1.

Exercise 4. Find the remainders of 2^{2009} modulo 3, 5, 7, and 9.

Exercise 5. 2005 AMC 10A. Problem 16. The sum of the digits of a two-digit number is subtracted from the number. The units digit of the result is 6. How many two-digit numbers have this property?

Exercise 6. Let m and n be integers. Prove that $mn(m+n)$ is even.

Exercise 7. Find all prime numbers p and q such that $p^2 - 2q^2 = 1$.

Exercise 8. Prove that there are 1000 consecutive composite numbers.

Exercise 9. Prove that 3, 5 and 7 is the only triple of twin primes.

Exercise 10. Prove that for $n > 2$, number $2^n - 1$ and $2^n + 1$ can't be both prime.

Exercise 11. Prove that two consecutive Fibonacci numbers are coprime.

Exercise 12. HMMT. Find the largest square of the form $1! + 2! + 3! + \ldots + n!$.