Remainders. Other Homework problems

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April 6, 2009

It was mentioned on CNN that the new prime number discovered recently is four times bigger then the previous record.

Finish the problems from the class handout.

Competition practice

Exercise 1. 2005 AMC 10A. Problem 21. For how many positive integers n does $1 + 2 + \cdots + n$ evenly divide $6n$?

Exercise 2. 2006 AMC 10B. Problem 11. What is the tens digit in the sum $7! + 8! + 9! + \cdots + 2006!$?

Exercise 3. 1986 AIME. What is that largest positive integer n for which $n^3 + 100$ is divisible by $n + 10$?

Exercise 4. 1984 AIME. The integer n is the smallest positive multiple of 15 such that every digit of n is either 8 or 0. Compute $n/15$.

Exercise 5. 1983 AIME. Let a_n equal $6^n + 8^n$. Determine the remainder upon dividing a_{83} by 49.

Challenge Problems

Exercise 6. I have some candy. I am expecting some guests and don't know how many can make it. I have counted my candy and noticed that if I divide the candy into two equal piles, then one piece of candy will be left over. If

I divide the candy into three equal piles, then 2 pieces of candy will be left over, if I divide it into 4 piles, then 3 pieces of candy will be left over. If I divide it into five piles, then 4 pieces of candy will be left over, and if I divide the candy into six piles, then 5 pieces of candy will be left over. How much candy do I have?

Exercise 7. Give an example of 5 numbers such that their product is not 0, and such that if you subtract 1 from each number, then their product will remain the same.

Exercise 8. 25 lazy students were standing on a straight line all facing the same direction that is perpendicular to the line. The Math Teacher said: "Everybody turn to the left" after which some students turned to the left, some turned to the right, and some didn't move. Is it always true that the Math Teacher can now insert herself into the line in such a way that the numbers of students facing her on both of her sides will be the same?

Exercise 9. How many binary words of length 11 are there such that every digit appears only an odd number of times in a row?