

Logic and Arithmetic

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Class Discussion

Speed counting

Warm Up

Exercise 1.

$$\frac{(7 - 6.35)/6.5 + 9.9}{(1.2/36 + 1.2/0.25 - 1\frac{5}{16})/\frac{169}{24}}$$

Exercise 2. Pooh poured himself a cup of coffee, then drank $1/2$ of the cup, then topped it off with milk, stirred it, then drank $1/3$ of the cup, then topped it off with milk, stirred it, then drank $1/6$ of the cup, then topped it off with milk, stirred it, drank up the whole cup. Find the ratio of the amounts of coffee and milk that he had drunk.

Problem Set

Exercise 3. There are 4 people in a family. If the daughter's salary is doubled, then the total family income will increase by 5%. If, instead, Mom's salary is doubled, it will increase by 20%. If Dad's salary is doubled, it will increase by 20% too. By what percentage will the total income increase if Grandpa's pension is doubled?

Exercise 4. 2002 AMC 10A, Problem 12. Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages 40

miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

Exercise 5. On an island, $\frac{2}{3}$ of all men are married, and $\frac{3}{5}$ of all women are married. What percentage of the island's population is married?

Exercise 6. There are 500 pennies in Pooh's piggy bank. The piggy bank is malfunctioning. As a result Pooh has only 2 options: either take out exactly 300 pennies, or deposit exactly 198 pennies, but he can perform these transactions as many times as he needs. What is the maximum amount of money Pooh can take out without braking his piggy bank?

Exercise 7. While trick-o-treating, Pooh collected a total of 30 items. He noticed that no matter how he selected 12 of these items, there was always at least 1 truffle among them. He also noticed that if he selected any 20 of his items, then at least one of those would be a bubble gum. How many truffles did he have?

Exercise 8. A man finishes work early, and so he takes the early train home, arriving at the train station one hour earlier than normal. He starts walking home and walks at a constant speed. The same day his wife decided to give her husband a surprise and not knowing that he arrived earlier, she drives to the train station to pick him up at the normal time. She drives the car with a constant speed and plans to be at the station exactly on time. Luckily for both of them he walks home the same way she drives (in the opposite direction, of course). His wife sees him on the road, and picks him up. They arrive home 20 minutes earlier than he would have arrived by himself. How long was the man walking?

Exercise 9. Given 99 blank cards, we first write the numbers 1, 2, ..., 99 on them, one number per card. We then shuffle the cards, turn them upside down, and again write the numbers 1, 2, ..., 99 on them. Then, for each card, we add the 2 numbers written on it, and then multiply these 99 new numbers. Prove that the result is an even number.

Exercise 10. A farmer brought 9 cows to a pasture and they ate all the grass in 6 days. If he had brought 8 cows instead, they would have eaten all the grass in 9 days. What's the maximum number of cows that can feed on this pasture forever, while the grass is growing?