Induction

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Class Discussion

Mathematical Induction.

Warm Up

Exercise 1. A stick has two ends. If you cut off one end, how many ends will the stick have left?

Exercise 2. A hypotenuse of a right triangle is 10 inches, and the altitude having the hypotenuse as its base is 6 inches. Find the area of the triangle.

Exercise 3. What is the smallest prime divisor of $5^{2009} + 1$?

Problem Set

Exercise 4. Use the mathematical induction to prove that the Fibonacci sequence F_n satisfies $\sum_{i=0}^{n} F_i = F_{n+2} - 1$.

Exercise 5. Use the mathematical induction to prove that the Fibonacci sequence F_n satisfies $F_{n+1}^2 = F_n F_{n+2} + (-1)^n$.

Exercise 6. For integers a and b, prove that if $a^2 + b^2$ is divisible by 3, then it is divisible by 9.

Exercise 7. Use the mathematical induction to prove that $1 + 3 + 5 + ... + (2n - 1) = n^2$.

Exercise 8. How many zeroes at the end does 100! have?

Exercise 9. Use the mathematical induction to prove that $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$.

Exercise 10. USSR bank has an unlimited number of 3-ruble and 5-ruble bills. Prove that it can pay any number of rubles starting from 8 exactly.

Exercise 11. USAMO 2003. Use the math induction to prove that for every positive integer n there exists an n-digit number divisible by 5^n all of whose digits are odd.

Exercise 12. Assume you have a chocolate bar consisting, as usual, of a number of squares arranged in a rectangular pattern. Your task is to split the bar into small squares (always breaking along the lines between the squares) with a minimum number of breaks. How many will it take?