Combinatorics 2

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Question: How do you keep a programmer in the shower all day?
Answer: Give him a bottle of shampoo which says "lather, rinse, repeat."

Class Discussion

If you like programming check out TopCoder: www.topcoder.com. High school competitions: http://www.topcoder.com/tc?module=Static&d1=hs&d2=home. If you register you can use my handle **axchma** as a reference.

Combinatorial identities:

$$\binom{n}{k} = \binom{n}{n-k},$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k},$$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n.$$

Pascal's triangle.

Warm Up

Exercise 1. How many 3-digit numbers are divisible by 7?

Exercise 2. Tanya's math club has 17 boys, and 23 of the students program in Java. If 14 of the students that program in Java are girls, then how many boys do not program in Java?

Problem Set

Exercise 3. What is the units digit of the sum: $1! + 2! + 3! + 4! + \ldots + 100!$?

Exercise 4. How many 4-digit numbers have exactly one zero?

Exercise 5. How many 5-digit palindromes are there?

Exercise 6. In how many ways can you rearrange letters of my last name — Khovanova?

Exercise 7. In Tanya's math class of 10 people, 4 do not know how to play set. Tanya wants to make 2 teams of five people each to play set in such a way that every team has at least one person who knows how to play and at least one person who doesn't. In how many ways can she do that?

Exercise 8. You have infinitely many playing card decks. In how many ways can you have a hand of three kings?

Exercise 9. Tanya's secretary needs to send out 3 invitation letters for Tanya's math party. In a hurry she messed up the letters and the envelopes. Every letter was mailed to a wrong person. In how many ways could the secretary have messed up? Solve the same problem for 4 letters.

Exercise 10. Simplify:

$$\frac{\sqrt{x}+1}{x\sqrt{x}+x+\sqrt{x}} \Big/ \frac{1}{x^2-\sqrt{x}}.$$

Exercise 11. (AIME) An integer is called *snakelike* if its decimal representation $a_1a_2a_3\cdots a_k$ satisfies $a_i < a_{i+1}$ if i is odd and $a_i > a_{i+1}$ if i is even. How many snakelike integers between 1000 and 9999 have four distinct digits?