

Combinatorics 2. Other Homework problems

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Finish the problems from the class handout.

Competition practice

Exercise 1. 2003 AMC 10A, Problem 21. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on a tray. How many different assortments of six cookies can be selected?

Exercise 2. 2003 AMC 10B, Problem 10. Nebraska, the home of AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by 4 digits. Each new license plate consists of three letters followed by three digits. By how many times is the number of license plates increased?

Exercise 3. 2003 AMC 10B, Problem 16. A restaurant offers three deserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a desert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2003, which is not a leap year?

Exercise 4. 2004 AMC 10A, Problem 12. Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties and any collection of condiments. How many kinds of hamburgers can be ordered?

Challenge Problems

Exercise 5. Tanya's secretary needs to send out 5 invitation letters for Tanya's math party. In a hurry she messed up the letters and the envelopes. Every letter was mailed to a wrong person. In how many ways could the secretary have messed up?

Exercise 6. In how many ways can you tile a 2 by 10 rectangle by 2 x 1 dominoes?

Exercise 7. Finish solving mystery hunt puzzle Functions given the functions:

- A. The output is $x/2$.
- B. The output is the product of the digits of x .
- C. The output is x divided by the last digit of x .
- D. The output is the number of letters in the English name of the number x .
- E. The output is the number of factors of x .
- F. The output is the sum of the digits of x .
- G. The output is the square root of x , rounded down.
- H. This is Euler's totient function, a.k.a. the Euler Phi function. The output is the number of positive integers less than or equal to x which are relatively prime to x .
- I. The output is 8.
- J. This function takes the n th triangular number to n . To be precise, the output is $(\sqrt{1+8x}-1)/2$.
- K. The output is the number of 1's in the binary representation of x .
- L. The output is the x th decimal digit of pi, treating the "3" before the decimal point as the first digit.
- M. The output is the first letter in the English name of the number x , converted to a number in the range 1-26 in the usual way.
- N. The output is the natural log of x , rounded down.
- O. The output is the number of distinct prime factors of x .
- P. The output is the mean of the digits of x .
- Q. The output is the number of letters in the Roman numeral representation of x .
- R. This function performs the Fahrenheit to Celsius conversion on x . The output is $5/9*(x-32)$.