

Divisibility

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April 17, 2008

Warm Up

Exercise 1. Mike and Tom went to a yard sale and wanted to buy a Yoda toy. Mike needed 10 more cents to buy the toy and Tom needed 1 more cent. They put their money together and they still didn't have enough. How much was Yoda?

Exercise 2. Bob has two more sisters than brothers. How many more daughters than sons do Bob's parents have?

Problem Set

Exercise 3. Can you replace the stars in the equation $1 * 2 * 3 * \dots * 10 = 0$ with pluses and minuses to get a correct equality?

Exercise 4. Prove that the number of different divisors of n (including 1 and n) is odd if and only if n is a square.

Exercise 5. A number is written with 300 ones and all other digits are zeroes. Can this number be a square?

Exercise 6. A two digit number is summed up with its reverse. The resulting number is a square. Find all such numbers.

Exercise 7. Prove that the product of four consecutive integers plus one is a square.

Exercise 8. Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.

Exercise 9. Prove that $7^{2n} - 5^{2n}$ is divisible by 24.

Exercise 10. How many zeroes does $100!$ have at the end?

Exercise 11. For what primes p does there exist a number consisting only of ones that is divisible by p .

Exercise 12. Prove that for any n the number written with 3^n ones is divisible by 3^n .